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CASE STUDY

A Mathematical Model of Marine Diesel Engine Speed Control System

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Abstract Diesel engine is inherently an unstable machine and requires a reliable control system to regulate its speed for safe and efficient operation. Also, the diesel engine may operate at fixed or variable speeds depending upon user's needs and accordingly the speed control system should have essential features to fulfil these requirements. This paper proposes a mathematical model of a marine diesel engine speed control system with droop governing function. The mathematical model includes static and dynamic characteristics of the control loop components. Model of static characteristic of the rotating fly weights speed sensing element provides an insight into the speed droop features of the speed controller. Because of big size and large time delay, the turbo charged diesel engine is represented as a first order system or sometimes even simplified to a pure integrator with constant gain which is considered acceptable in control literature. The proposed model is mathematically less complex and quick to use for preliminary analysis of the diesel engine speed controller performance.

Keywords Speed droop · Governor · Damping factor · Time delay

Notations

K_1	Speed sensing element gain constant
K_2	Hydraulic power amplifier gain constant
K_e	Turbocharged diesel engine gain constant
K_s	Speeder spring stiffness, N/m
L_1	Lever length of right, mm
L_2	Lever length of left, mm
N_e	Engine speed, rpm
N_{ref}	Engine set speed, rpm
T	Temperature of lubricating oil
T_1	Time constant of diesel engine combustion
T_2	Time constant of firing delay
a	Distance from flyweight fulcrum to flyweight centre
m	Mass of flyweight, kg
z	Speed sensor clutch displacement, mm
ω_n	Natural frequency, rad/s
μ	Viscosity of lubricating oil, m^2/s
ϕ	Pre-tension of spring, mm

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Introduction

Prime mover speed governing is essentially a control function and therefore its design, analysis and synthesis is based on standard control engineering principles. The speed controller (governor) design could be based either on proportional (P) or proportional plus integral (P + I) plus derivative (P + I + D) strategy depending upon the type of applications. The governors used in marine prime movers are mostly proportional types which are characterized by an offset error in speed from the zero load rpm which is also called speed droop. The main reason for using droop type speed controllers (governors) in marine application is the convenience these units offer in electrical load sharing among various machines when operated together in parallel [1]. A typical marine diesel engine control system employing hydraulic activation is shown in Fig. 1.

Despite best efforts, designers rarely succeed in developing accurate mathematical models of complex system like a diesel

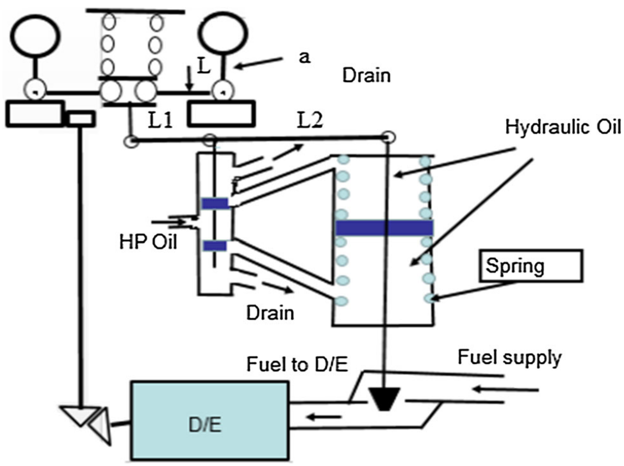

Fig. 1 Diesel engine speed control system

engine which can truly represent its dynamic characteristic. Additionally, wear of engine components with age and also deviations in operating conditions from rated point change the values of model coefficients which alter the dynamic characteristics and droop settings of the system. It is because of these limitations that speed governing of modern diesel engines is often implemented through flexible adaptive control structures based on simple engine models [2]. In this paper, a generic mathematical model of the engine has been projected by splitting into following four main components.

- Mechanical rotating fly weight speed sensing element;
- Hydraulic power amplifying unit;
- Diesel engine kinematics that include all mechanical moving parts;
- Combustion and fuel injection system including fuel control valve.

Since the objective of this paper is limited to the analysis of only the speed control function, a detailed mathematical modelling has been carried out only for the speed sensing element and hydraulic power amplifier. The kinematics and engine combustion process have not been modelled in greater detail. The diesel engine is treated as a first order system with time delay and constant gain. The generic model is intended to be used for developing reliable and robust speed control system using more complex controller algorithms [3].

Methodology: Mathematical Model of Static Characteristics

Speed Sensing Element

Despite great developments in electronics and digital technology, rotational speed of mechanical prime movers are still measured by using rotating fly weights, primarily because of their excellent safety and reliability records. Figure 2 shows a typical rotating fly weight mechanism which is directly connected to the prime mover fuel control valve for controlling its speed. Since for large prime movers, where fuel control valves are heavy and cannot be operated by the small force produced by the rotating masses, a hydraulic power amplifying unit is connected in series to provide extra needed power. The static characteristic of the rotating fly weight speed sensing element is explained in Fig. 2. When the prime mover is stationary, the action of the restoring spring keeps the fly weights collapsed in fully inward position.

But as prime mover begins to rotate, the fly weights move outwards due to the action of centrifugal force and tend to lift the clutch plate against the compression of the restoring spring. Hence, the downward acting spring force is always countered by the upward acting force generated from centrifugal action of the fly weights. The magnitude of the spring restoring force, E depends upon the amount of pre-strain, φ and the lift Z of the clutch plate. This can be expressed as [4],

$$E = f(\varphi, Z) \tag{1}$$

From Eq. (1), the change in restoring force can be expressed as,

$$\Delta E = \frac{\delta E}{\delta \varphi}\Delta\varphi + \frac{\delta E}{\delta z}\Delta z \tag{2}$$

Equation (2) establishes a static relationship between restoring force E and lift Z of the clutch plate at different speed of the prime mover and is called the characteristic equation of rotating fly weight speed sensing element. Figure 3 shows plots of the rotating fly weights speed sensor characteristics for different values of pre-strain φ in the restoring spring.

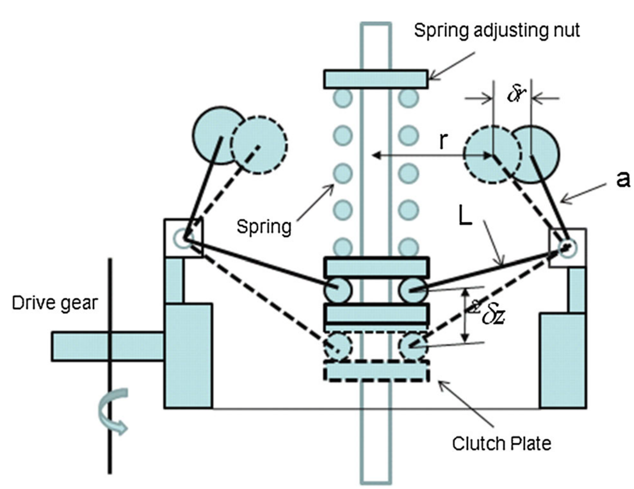

Fig. 2 Mechanical sensing element

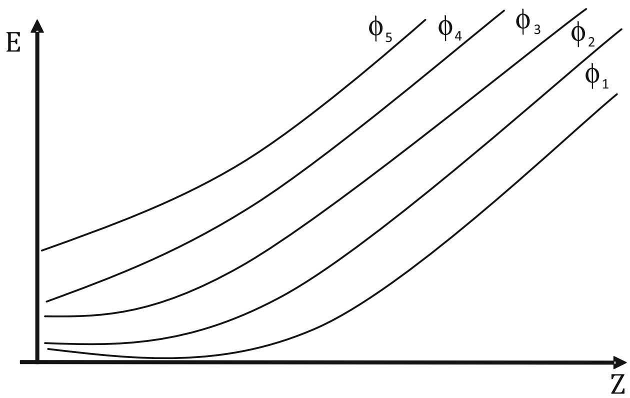

Fig. 3 Restoring force characteristics for varying pre-strain

Proceeding on a similar approach, the expression for the supporting force generated by centrifugal action of the rotating fly weights can be derived. For the purpose of analysis it is assumed that the prime mover is running at steady rpm with the angular velocity of fly weights at ω radians per second and the centre of gravity of each fly weight displaced by distance 'r' from the axis of rotation. The centrifugal force (CF) produced by each rotating fly weight is then expressed as in [2]

$$CF = m\, r\omega^2 \tag{3}$$

Referring to Fig. 2, the upward component of CF acting on the clutch plate which counters restoring force E can be then expressed as [4]

$$E = 2(a/L)(m\, r\omega^2) = A\omega^2 \tag{4}$$

where $A = 2\,(a/L)\,(mr)$.

Now consider a very small load variation of the prime mover resulting in a corresponding small change in the angular velocity. This will cause the fly weights to move away by a further distance of δr from the axis of rotation. This small additional radial change δr in the fly weight position causes small compression of the restoring spring resulting in an upward movement of the clutch plate by δz. As the stability of sensing element has been finally achieved at this new point, a mathematical expression can be derived by equating work done by the movement of fly weights and compression of restoring spring.

$$E\delta Z = 2(mr\omega^2)\delta r \tag{5}$$

Substituting for E from Eq. (4) in Eq. (5) as, $A\omega^2\delta Z = 2(mr\omega^2)\delta r$ it is rearranged to

$$A = 2(mr)\delta r/\delta z \tag{6}$$

From Eq. (6), it can be concluded that $A = f(z)$. Hence, from Eq. (4), the values of $A\omega^2$ can be plotted for varying z values. A plot of the supporting forces $A\omega^2$ and clutch lift

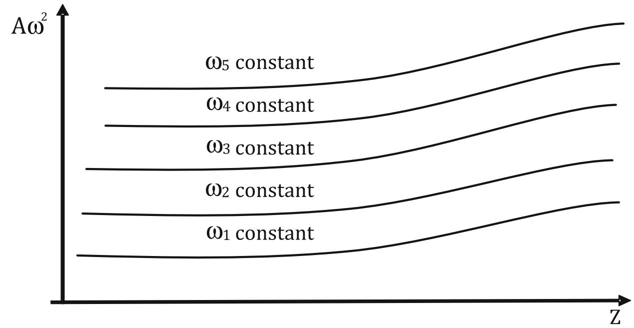

Fig. 4 Restoring force characteristics for varying angular velocities

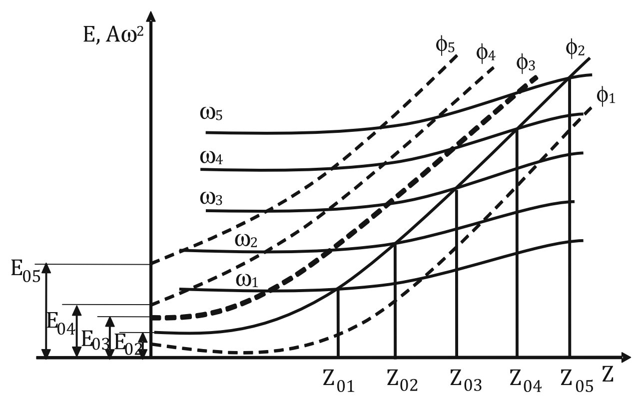

Fig. 5 Speed sensing element in equilibrium position

Z for different values of angular speed ω of the fly weights is shown in Fig. 4.

The plots of Figs. 3 and 4 can be combined to obtain steady state positions z of the clutch plate for different values of ω or corresponding engine speed ω_e as shown in Fig. 5 where restoring force corresponds to a specific pre-strain setting φ and intersects with corresponding CF force curve (ω = constant) at different points. Projections of these intersection points on the horizontal axis gives steady state positions z_0 of the clutch for the particular pre-strain setting of the restoring spring. Accordingly a unique set of clutch positions $z_{01} - z_{05}$ and fly weight speeds $\omega_{01} - \omega_{05}$ is obtained for the full operating range of the diesel prime mover.

The plot in Fig. 6a shows fly weight speed sensing element static characteristic for the constant pre-strained spring setting of ω. In the case of variable speed governors, the desired variable operating speed of the diesel prime mover is obtained by changing the pre-strain setting of the restoring spring during operation. Accordingly, a set of steady state clutch positions z_0 for each pre-strain setting of the restoring spring is obtained corresponding to different speed ω of the fly weights. Plots of variable speed governor characteristic curves for a set of pre-strained

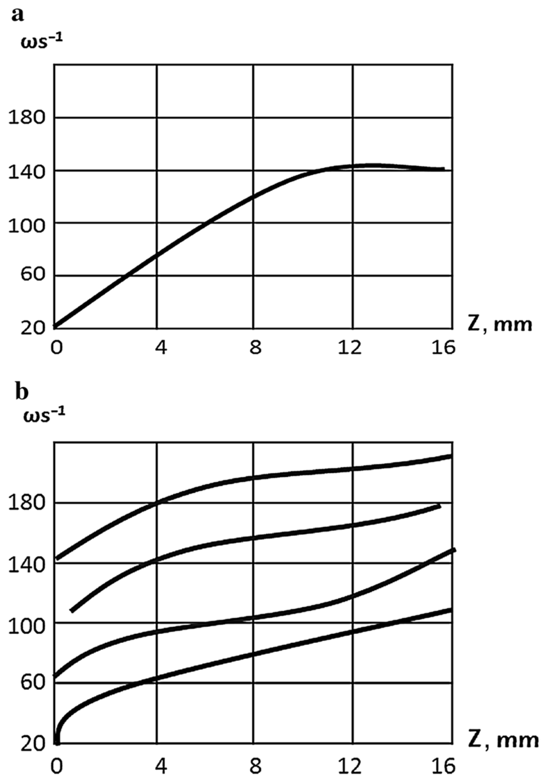

Fig. 6 Speed sensing element **a** static characteristics with constant pre-strain, ϕ and **b** static characteristics variable pre-strain, ϕ

spring is shown in Fig. 6b. These characteristic curves are used to determine speed droop of the governor.

Diesel Engine Load-speed Characteristics

Figure 7 shows the load-speed characteristic of the diesel engine. To obtain the load-speed characteristics of the governor, the steady state characteristics of the speed sensing element Fig. 6a and the load-speed characteristics of the diesel prime mover Fig. 7 can be matched. This is shown in Fig. 8. The decrease in diesel prime mover speed with increase in load and vice versa is indicated by the load-speed characteristic of the governor unit.

The drop in prime mover speed from no load to full load condition in Fig. 8, is the speed droop of the hydraulic governor unit and this depends on the rotating fly weight speed sensing element's static characteristic.

Dynamic Characteristics

Dynamic characterization of the prime mover and speed control mechanism is a key step in the analysis of engine overall performance during operation. Dynamic models of engine and control components are explained here.

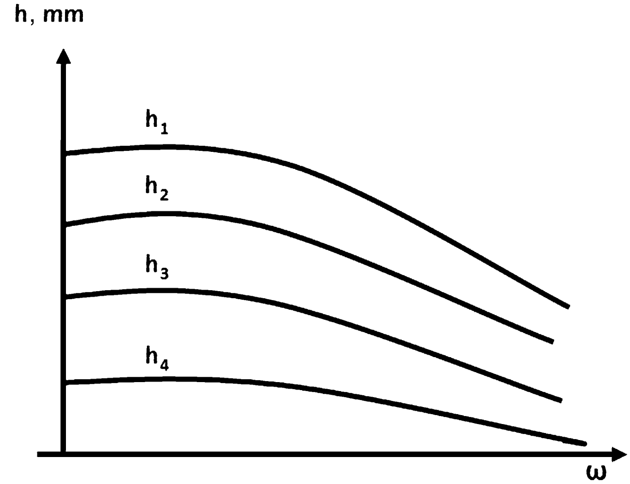

Fig. 7 Diesel engine load-speed characteristic

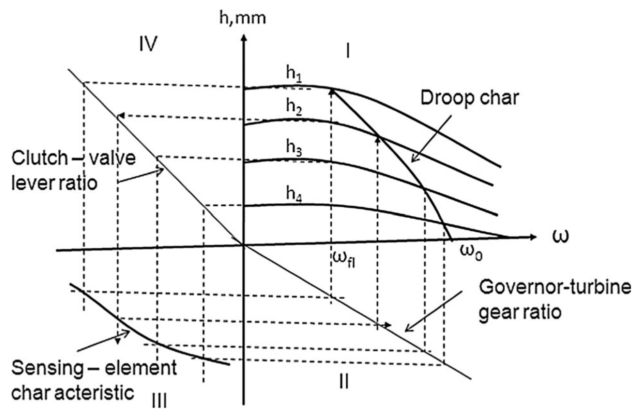

Fig. 8 Governor load-speed characteristic

Rotating Fly Weight Speed Sensing Element

This mechanism is essentially a basic spring mass damper system which is modelled from fundamental principles [5]. Referring to Fig. 2 the dynamics of rotating fly weight system [5] can be written as,

$$\frac{z(s)}{\omega_w(s)} = \frac{K_1}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{7}$$

Equation (7) can be represented in a block diagram as shown in Fig. 9.

Hydraulic Power Servo Amplifier

Marine prime movers are large high power producing machines and require large pulling force to operate their fuel valves. As the fly weights of the speed sensing element are small the pulling force produced by the centrifugal action of rotating masses is also small and not sufficient to operate the heavy fuel control valve. In such prime movers

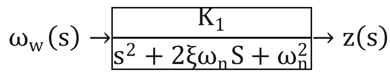

Fig. 9 Block diagram of speed sensing element where damping factor $\xi = f(m, \mu, T)$ and natural frequency $\omega_n = f(m\,k_s)$

the rotating fly weight speed sensing device is connected in series to a hydraulic power amplifying unit, as shown in Fig. 1. The power amplifier when in open loop configuration is a simple hydraulic integrator but if connected to the spool valve unit with a position feedback link (Fig. 1) then the combined unit results into a proportional controller [5]. The power cylinder position output y(s) will therefore be proportional to the input z(s). The term K_2 is the linear proportionality constant of oil flow with respect to spool valve displacement and L_1, L_2 are lever ratios as in Fig. 1. The block diagram in Fig. 10 shows the simplified block diagram of hydraulic power servo amplifier.

Diesel Engine Prime Mover

Turbocharged diesel-engine is a complex machine with many inherent delays, nonlinearities, and time lags because of which developing its accurate mathematical model is not always possible. However, despite this difficulty many linear as well as non-linear models of the diesel engines are available in the literature which have been mainly used for engine performance improvements [6, 7]. But fortunately for the purpose of control system design and analysis a very accurate mathematical model is not really necessary and most often a black box type model serves the purpose [8–10].

Herein, a turbocharged diesel engine has been modelled as a first order system with a time delay, shown in Fig. 11. The constant coefficient term K_e can be determined empirically from operating parameters. To investigate

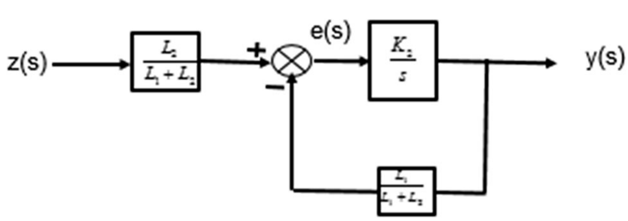

Fig. 10 Block diagram of hydraulic power servo amplifier

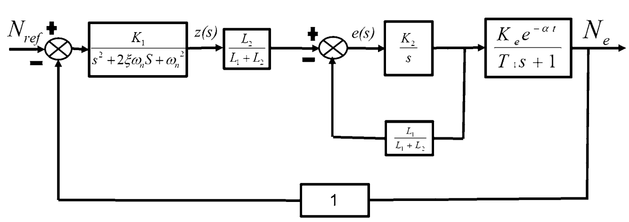

Fig. 11 Overall diesel engine governing system [5]

performance of the diesel engine speed control system, mathematical models of diesel engine, speed sensing element and hydraulic power amplifying unit are linked together as in Fig. 11. Assuming drive gear ratio as unity, the reduced open loop transfer function of the combined system clearly indicates that the overall system is type zero fourth order and hence its speed response to a step load change will be oscillatory during the transient period. However, this will finally settle to steady state with an offset error to characterize droop governing.

The model can be represented by the equation

$$\frac{N_e(s)}{N_{ref}(s) - N_e(s)} = \frac{K_1 K_2 K_e L_2}{(s^2 + 2\xi\omega_n s + \omega_n^2)\{(L_1 + L_2)s + k_2 L_1\}\{T_1 T_2 s^2 + (T_1 + T_2)s + 1\}} \tag{8}$$

The design data of all constants in Eq. (8) can be acquired from governor and engine manufacturers and used to simulate the system using either MATLAB or any other suitable software code. Alternatively the block diagram of Fig. 11 can be also simulated using MATLAB-SIMULINK for performance analysis.

Simulation Study

As the firing order time delay in multi-cylinder high speed diesel engines is very small, the engine can be modelled as a first order system without causing appreciable error in the overall result [8]. Hydraulic servo is also an approximation to a pure integrator. Taking these two factors into account, the open loop transfer function of the diesel engine combined with governor becomes a fifth order system as shown in Eq. (8). As no real engine data could be obtained from the engine manufacturer, simulation study on the revised model was carried out using data from [8] and the engine data are tabulated in Table 1.

Results and Discussion

Data from [8] has been used in the simulation to test the mathematical modelling approach. On substituting engine data, the coefficient of the fifth order term in the transfer

Table 1 Engine data

Characteristic	Value
Type	4-stroke, 16 cylinder, V-type, diesel engine
Speed	1200 rpm
Power	1500 kW
Fuel pump	Jerk type
Governor	Hydraulic, variable speed with droop

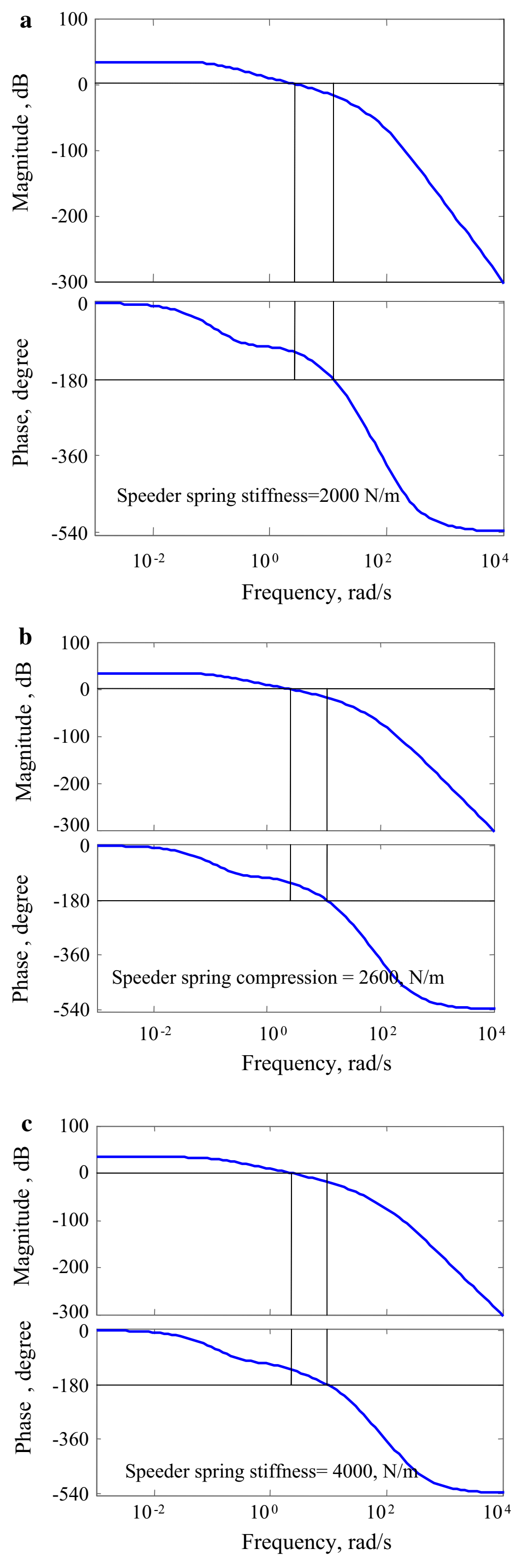

Fig. 12 Bode plot of engine-governor model: Bode diagram **a** Gm = 17.9 dB (at 12.5 rad/s), Pm = 63.9° (at 2.66 rad/s); **b** Gm = 18.5 dB (at 11.2 rad/s), Pm = 59° (at 2.53 rad/s); **c** Gm = 19.5 dB (at 9.43 rad/s), Pm = 50.6° (at 2.28 rad/s)

function was found very small and so this term was neglected leaving the plant model a type-zero fourth order system. The stability of the engine-governor model has been investigated through frequency response and root locus plots against variation of open loop gain [5]. Open loop gain was varied by altering the stiffness of the speeder spring in the range from 1300 to 3500 N/m.

Bode plot results in Fig. 12a–c indicate gain and phase margin of the governor speed control system improves while increasing the speeder spring compression from 2000 to 4000 N/m. This is expected as increase in the speeder spring compression reduces system open loop gain and thus gives a higher value of the gain and phase margins.

This result was also verified through root locus plot. Locations of open loop poles and zeroes indicate that the two dominant poles lie in the negative half S-plane, close to the origin. The remaining two complex poles have large negative real parts, reflective of a fast oscillatory decay of the engine transient response. This is verified with the computed step responses in MATLAB SIMULINK as shown in Fig. 13a–d.

Increasing spring stiffness from 1300 to 3500 N/m resulted in complex poles moving away from the real axis, indicative of increasing oscillatory response. As seen from Fig. 13a–d the computed and simulated responses are consistent. However, for all cases, the response is seen to settle within a reasonable period of 4 to 7 s and compares well with real generator response time during sudden load change as projected in Gant [3].

The proposed mathematical model can be used to simulate engine speed control performance, both, in mechanical as well as electrical load applications. The validation of the model is further elaborated by studying the case of two shipboard generators running in parallel load sharing operation. Both generators are three-phase, 440 V 60 Hz rated at 1500 kW running at 1200 rpm and run by diesel engines.

In such operating modes, differing droop characteristics of speed controllers make individual generator units load share inversely proportional to their droop [11–14]. The load sharing among different diesel generators (D/G) fitted with droop type governors and connected in parallel mode to a mini-grid will involve investigation of real as well as reactive powers so that the effect of droop change on voltage variation can be observed. However, to simplicity simulation in this research, the Auto Voltage Regulator (AVR) is assumed to be fully decoupled from the frequency control system and the generator voltage remains constant. The results of real power sharing between both generators from Gant [8] are shown in Table 2. Herein, the Open Loop Gain has been varied by only altering speeder spring constant but it also depends on many other factors

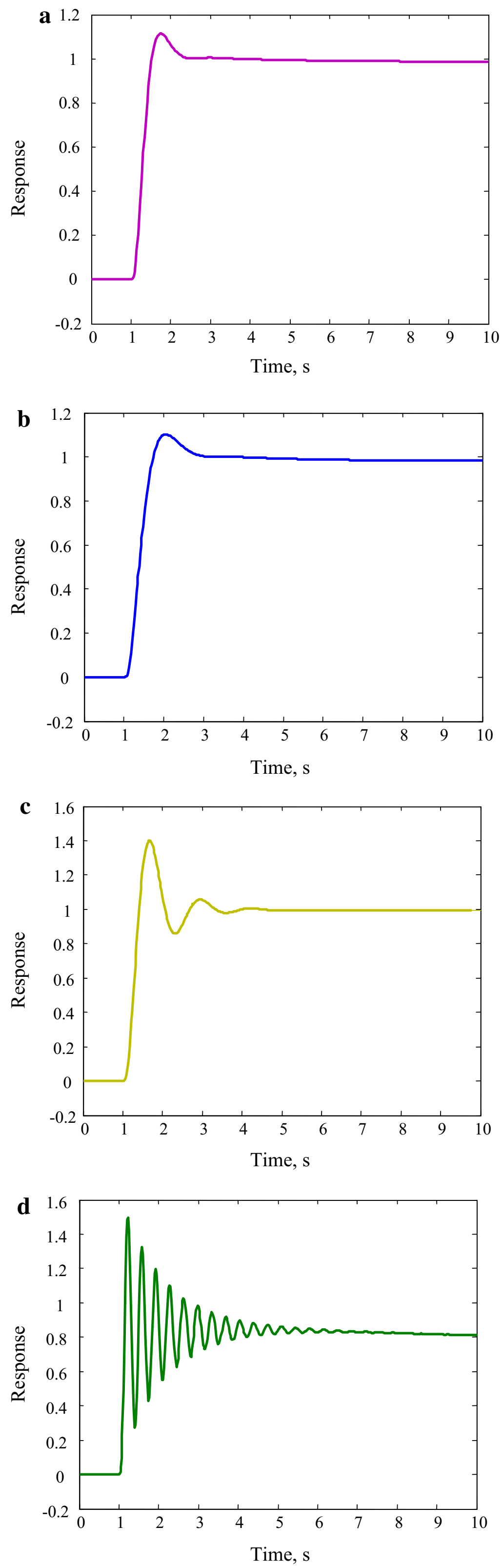

Fig. 13 Responses from simulations in SIMULINK: time series plot: **a** Beta 1300 N/m; **b** Beta 2000 N/m; **c** Beta 2600 N/m; **d** Beta 3500 N/m

Table 2 Real power sharing of diesel generators with different droop [8]

Unit	Gain	Droop, %	Shared load, %
D/G 1	14.0	6.6	56.70
D/G 2	10.5	8.4	43.30
D/G 1	17.5	5.5	61
D/G 2	10.5	8.4	39

like governor drive gear ratio, lubricating oil viscosity, governor power piston lever ratio etc. Generally shipboard generators operate within droop limits of 5 to 6% and governors of main propulsion diesel engines operate within 8 to 10%. This range of droop setting could be achieved during simulation by setting gain values in the range of 10 to 15 as seen from the results. The gain setting of 25 has been used by Gant [8] with 4% droop, and in addition to speeder spring setting, changes in oil viscosity also has been considered.

The result in Table 2 shows both generators share load in the inverse proportions to their individual droop and as expected run at the same speed. The actual speed difference between both generators was found to be less than 1 rpm at this loading condition which is well within speed measuring errors of shipboard power generating units. Load sharing result matches with actual behaviour of shipboard generators during parallel running.

Conclusion

Mathematical models of a speed control system of marine turbocharged diesel engine has been described from the first principle. Both the static as well as dynamic characteristics of the speed sensing and hydraulic power unit have been modelled. The turbocharged diesel engine has been modelled as a simple first order system with time delay, which is considered satisfactory for the purpose of speed control analysis of a large diesel engine.

The proposed model is simple and handy for preliminary investigation of diesel engine speed control system performance. The static characteristic model of the rotating fly weights (speed sensing element) gives a clear insight into the droop characteristics of speed controller and also the variable speed operation of the engine. Simulation results from the proposed mathematical model show consistent performance.

References

1. T. Hellstorm, Optimal pitch, speed and fuel control at sea. J. Mar. Sci. Technol. **12**(2), 71–77 (2004)

2. R.P. Sinha, Robust adaptive control of marine diesel propulsion system, in *Proceedings of International Conference in Mechanical & Manufacturing Engineering*, 20–22 May 2008, Johor Bahru (2008)
3. G. Baker, *PID Tuning of Plants with Time Delay Using Root Locus, Master's Thesis and Graduate Research*. San Jose State University, SJSU Scholar Works (2011)
4. V.I. Krutov, *Automatic Control of Internal Combustion Engines* (Mir Publishers, Moscow, 1987)
5. K. Ogata, *Modern Control Engineering*, 5th edn. (Prentice-Hall, Pearson Education, Englewood Cliffs, London, 2013)
6. P. Kucera, V. Pistek, Virtual diesel engine in SIMULINK. University of Technology Brno, Faculty of Mechanical Engineering. Institute of Automotive Engineering. Technika 2896/2 8(2), 95–105 (2013)
7. Y.H. Zweiri, J F. Whidborne, F.D. Seneviratne, Complete Analytical Model of a single Cylinder Diesel Engine for Non-linear Control and Estimation, Report 99/10. Department of Mechanical engineering, King's College, London UK (1999)
8. G.C. Gant, The governing of diesel engines, in *Principles and Performance in Diesel Engineering*, ed. by S.D. Haddad, N. Watson (Ellis Horwood, Chichester, 1984)
9. F. Zhao, W. Yang, W.W. Tan, S.K. Chou, W. Yu, An overall ship propulsion model for fuel efficiency study. 7th international conference on applied energy ICAE 2015. Energy Procedia **75**, 813–818 (2015). https://doi.org/10.1016/j.egypro.2015.07.139
10. A.T. Karlsen, *On Modeling of a Ship Propulsion System for Control Purposes*. MS Thesis, Norwazian University of Science and Technology, Department of Engineering Cybernetics (2012)
11. J.M. Prousalidis, E. Xanthapoulos, K. Voutzolidis, Reactive power sharing in ship energy systems with shaft generators. J. Mar. Eng. Technol. **8**(1), 21–38 (2009). https://doi.org/10.1080/20464177.2009.11020216
12. A.M.R. Ibrahim, H.A. Ashour, M.M. El Attar, Load sharing realization of parallel operated synchronous generators with in ship micro-grid using microcontrollers, in *The International Maritime Transport & Logistic Conference (MARLOG 2) Sustainable Development of Suez Canal Region,* 17–19 March 2013
13. A.M. Bollman, *An Experimental Study of Frequency Droop in a Low Inertia Micro-grid*, MS Thesis, Electrical and Computer Engineering, University of Illinois, Arbana Champaign, USA (2009)
14. G. Olson, Paralleling Dissimilar Generators Part-3, Load Sharing Compatibility, Power Topic 9017. Technical Information from Cummins Power Generation (2010)