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INDIAN MARITIME UNIVERSITY
(A Central University, Govt. of India)

B.Sc. (Nautical Science) - Semester – I
December 2015 End Semester Examinations

APPLIED MATHEMATICS - I
Subject Code: UG21T2104

Time : 3 hrs | Max.Marks :70
Date: 14.12.2015 | Pass Marks: 35

NOTE: Attempt any FIVE questions out of 7. All questions carry equal marks.

1. a) Find the complex number z if $arg(z+1) = \frac{\pi}{6}$ and $arg(z-1) = \frac{2\pi}{3}$.

b) Prove that $(1+i)^n + (1-i)^n = (2)^{\frac{n}{2}+1} \cos\frac{n\pi}{4}$.

c) If $y = \log(\tan x)$ show that,

(i) $\sinh ny = \frac{1}{2}[tan^n x - cot^n x]$, (ii) $\cosh(n+1)y + \cosh(n-1)y = 2\cosh ny \,.\, cosec\ 2x$.

(4+5+5 marks)

2. a) If ω is a complex cube root of unity,prove that $1 + \omega + \omega^2 = 0$.

b) If $\tan\frac{x}{2} = \tanh\frac{u}{2}$ prove that,

(i) $\sinh u = \tan x$ (ii) $\cosh u = \sec x$ (iii) $u = log_e\left[\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right]$. (7+7 marks)

3. a) Prove that $sech^{-1}(\sin\theta) = \log\cot\frac{\theta}{2}$.

b) If $\tan\log(x+iy) = a + ib$ where $a^2 + b^2 \neq 1$, show that $\tan\log(x^2+y^2) = \frac{2a}{1-a^2-b^2}$.

(7+7marks)

4. a) If $y = x\log\frac{x-1}{x+1}$, show that $y_n = (-1)^{n-2}(n-2)!\left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$.

b) If $u = tan^{-1}\frac{x^3+y^3}{x+y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

and $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$.

(7+7 marks)

5. a) If $u = tan^{-1}\left(\frac{y}{x}\right)$ where $x = e^t - e^{-t}$ and $y = e^t + e^{-t}$ find $\frac{du}{dt}$.

b) The period of a simple pendulum is $T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$, find the maximum error in T due to the possible error upto 1% in l and 2.5% in g.

c) Prove that $\frac{b-a}{b} < \log\left(\frac{b}{a}\right) < \frac{b-a}{a}$ for $0 < a < b$. (4+5+5 marks)

6. a) If $f(x,y) = 0$ and $\varphi(x,z) = 0$ then show that $\frac{\partial\varphi}{\partial x}.\frac{\partial f}{\partial y}.\frac{dy}{dz} = \frac{\partial f}{\partial x}.\frac{\partial\varphi}{\partial z}$.

b) Show that the minimum value of $u = xy + a^3\left(\frac{1}{x} + \frac{1}{y}\right)$ is $3a^2$. (7+7marks)

7. a) If $v = \log(x^2 + y^2 + z^2)$,prove that $(x^2 + y^2 + z^2)\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] = 2$.

b) By using Maclaurin's series expand $\log(1 + e^x)$ in powers of x upto x^4.

(7+7 marks)

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