Indian Maritime University

(A Central University, Govt of India)

End Semester Examinations – December 2024

Programme Name: B Tech (ME)

Semester: First

Subject Code: UG11T5102

Subject Name: Engineering Mathematics 1

Date: 12.12.2024	Max Marks: 70
Duration: 03 Hrs	Pass Marks: 35

Section A **(10X1=10 Marks)**

Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable.

1. The system of equations represented by a matrix AX = B is inconsistent if

   A. R(A) is not equal to R(A:B)

   B. R(A) = R(A:B)

   C. R(A) < R(A:B)

   D. R(A) > R(A:B)

2. Consider the following matrix A. Which is the correct statement.

$$A=\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 1\end{bmatrix}$$

   A. The matrix A is not a diagonal matrix.
   B. The eigen values of matrix are 1, 2 and 4
   C. The eigen values of matrix are 1 and 2
   D. None of the other options

3.Let X and y be two arbitrary, 3x3 nin -zero, skew -symmetric matrices and Z be an arbitrary 3x3 non-zero symmetric matrix, Then which of the following matrices is skew symmetric?

a) $y^3z^4 - z^4y^3$ b) $x^{44} + y^{44}$ c) $x^4z^3 - z^4x^3$ d) None of these

4) The curve passing through (0,1) and satisfying $\sin\left(\frac{dy}{dx}\right) = c'$ is

   a) $\cos\left\{\frac{(y-1)}{x}\right\} = c'$ b) $\sin\left\{\frac{(y-1)}{x}\right\} = c'$ c) $\cos\left\{\frac{x}{(y-1)}\right\} = c'$

   d) $\sin\left\{\frac{x}{(y-1)}\right\} = c'$

5) The complementary function of $(D^3 + D^2 - D - 1)y = 0$ is

   a) $c_1e^x - (c_2 + c_3x)e^{-x}$ b) $c_1e^{-x} + (c_2 + c_3x)e^x$ c) $c_1e^x + (c_2 + c_3x)e^{-x}$

   d) $c_1e^{-x} + (c_2 + c_3x)e^x$

6) **The number of arbitrary constants in the particular solution of a differential equation of third order is:**

   a) **3** **b) 2** **c) 1** **d) 0**

7)The number of non-zero rows in an echelon form is called?

a) rank of a matrix b) cofactor of the matrix c) reduced echelon form
d) conjugate of the matrix

8) The curve $x^3+y^3 = 3axy$ has symmetry about

   A. x-axis
   B. y-axis
   C. about line y=x
   D. about y=-x

9) The value of the line integral $\int_c (2xy^2dx + 2x^2ydy + dz)$ along a path joining the origin and the point (1,1,1) is

   a) 0 b) 2 c) 4 d) 6

10) . $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2)dxdy =$

   a) $\frac{3}{35}$ b) $\frac{7}{60}$ c) $\frac{4}{49}$ d) $\frac{2}{15}$

Section B

Five Questions of 02 Marks each (5X2=10 Marks)

11. Find nth order derivative of $y=e^{3x}\cos^2x$

12. Find rank of the given matrix by reducing to echelon form $A=\begin{bmatrix}0 & 1 & 2 & 1\\ 1 & 2 & 3 & 2\\ 3 & 1 & 1 & 3\end{bmatrix}$

13. Solve $\frac{dy}{dx} = e^{2x-3y} + 4x^2e^{-3y}$

14. A particle moves along the curve X=$t^3+1, y=t^2, z=2t+3$, where t is the time. Find the component of velocity and acceleration at t=1 in the direction $\vec{i}+\vec{j}+3\vec{K}$

15. If u=xy-yz-zx, v=$x^2+y^2+z^2$, $w=x+y-z$, determine whether they are functionally related or not, if so find the relationship between them.

Section C

Seven Questions of 10 Marks each of which any 05 questions to be answered.

16.a) If y=$(x^2-1)^n$, then show that

$$(x^2-1)y_{n+2}+2xy_{n+1}-n(n+1)y_n=0 \qquad \text{(5 marks)}$$

16. b) Using Lagranges's method of undetermined multipliers find the maximum and minimum distances of the point (3,4,12) from the sphere

$$x^2+y^2+z^2=4 \qquad \text{(5 marks)}$$

17.a) If u=$sin^{-1}\sqrt{\frac{x^{\frac{1}{3}}+y^{\frac{1}{3}}}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}}$; prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{12}\left(\frac{13}{12}+\frac{tan^2 u}{12}\right) \qquad \text{(5 marks)}$$

17.b) Find the directional derivative of f(x,y,z)=$4e^{2x-y+z}$ at the point

(1,1,-1) in the direction towards the point (-3,5,6) (5 marks)

18.a) The upward speed $v(t)$ of a rocket at time t is approximated by

$v(t) = at^2 + bt + c$, $0 \le t \le 100$ where a, b, and c are constants. It has been found that the speed at times t = 3, t = 6, and t = 9 seconds are respectively, 64, 133, and 208 miles per second. Find the speed at time t = 15 seconds. Solve the problem using Gaussian E limination. (6 marks)

18.b) Find the eigenvalues and eigenvectors of A=$\begin{bmatrix}2 & 1\\ 4 & -1\end{bmatrix}$ (4 marks)

19. a) Find the characteristics equation of the matrix A=$\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$ and hence find A^{-1} . Also, find the matrix represented by

$$A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I \qquad \text{(5 marks)}$$

(b) Test for consistency

$x+y+z=6$; $x+2y+3z=14$; $x+4y+7z=30$ and solve them.
(05)

20.a) Define Hermitian Matrix and Skew Hermitian Matrix.

If A=$\begin{bmatrix}2+i & 3 & -1+3i\\ -5 & i & 4-2i\end{bmatrix}$, show that AA* is Hermitian matrix,

where A* is the conjugate transpose of A.
(04)

b) Find the Eigen values and Eigen vectors for the matrix

$$A = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & -3 & 3\end{bmatrix}$$
(06)

21.a) If c is a simple closed curve in the xy-plane not enclosing the origin, show that using Green's theorem $\int_c F.dR=0$, where F=$\frac{y\,I-xJ}{X^2+Y^2}$ (5 marks)

21.b) Find the area between the parabolas $y^2=4ax$ and $x^2=4ay$.
(05)

22.a) Prove that $\vec{F}=(6xy+z^3)\vec{\imath}+(3x^2-z)\vec{\jmath}+(3xz^2-y)\vec{k}$ is irrational. Find a scalar function ∅ (x, y, z)such that $\vec{F}=\nabla\emptyset$.
(05)

b) Find the work done in moving the particle under the field of force given by $\vec{F}=(x^2-y^2+x)I-(2xy+y)J$ in the xy plane from the point (0, 0) to (1, 1) along the curve $y^2=x$.
(05)