

Indian Maritime University
(A Central University, Govt of India)
End Semester Examinations – December 2023
Programme Name: B Tech (NASB)
Semester: I
Subject Code: UG13T1101
Subject Name: Engineering Mathematics-I

Date: 15.12.2023

Max Marks: 70

Duration: 03 Hrs

Pass Marks: 35

General Instructions

- (i) All Sections (A, B & C) are to be attempted.
- (ii) Options, if any, are specified in respective section.

Section A

Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable.

1. The series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ is []

- a) Convergent
- b) Not Convergent
- c) Absolutely Convergent
- d) None of the above

2. The order of differential equation $\left(\frac{d^3y}{dx^3} + 1\right)^{\frac{1}{3}} = \left(\frac{d^2y}{dx^2}\right)^4$ []

- a) 1
- b) 5
- c) 3
- d) 2

3. If $u = x^3 + y^3$ then $\frac{\partial^2 u}{\partial x \partial y}$ is equal to []

- a) -3
- b) 3
- c) 0
- d) $3x + 3y$

4. If $2x - x^2 + ay^2$ is harmonic then $a =$ []

- a) 1
- b) 2
- c) 3
- d) 0

5. The C-R equations for $f(z) = u(x, y) + iv(x, y)$ to be analytic are _____

6. If $\sum u_n$ is a convergent series of positive terms then $\lim_{n \rightarrow \infty} u_n =$ []

- a) 1
- b) ± 1
- c) 0
- d) ∞

7. Particular Integral of $(D^2 + 4)y = \cos 2x$ is _____

8. The value of c of the Cauchy's mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ in $[2, 3]$ is _____

9. The value of definite integral $\int_{-a}^a |x| dx$ is equal to []

- a) a
- b) a^2
- c) 0
- d) $2a$

10. The value of $\int_0^1 \int_0^{x^2} x e^y dy dx$ is _____

Section B

Five Questions of 02 Marks each

11. Evaluate $\lim_{x \rightarrow 0} \frac{\log x}{\cot x}$

12. Solve $(D^3 + D^2 + 4D + 4)y = 0$.

13. Verify Cauchy's mean value theorem for functions e^x and e^{-x} in the interval (a, b)

14. Find the second partial derivatives of $z = x^3 + y^3 - 3ax$.

15. Test for convergence $\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$

Section C

Seven Questions of 10 Marks each of which any 05 questions to be answered.

16. Expand $\log_e x$ in powers of $(x - 1)$ and hence evaluate $\log_e 1.1$ correct to 4 decimals.

17. If $u = \sin^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \tan x$,

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{-\sin u \cos 2u}{4 \cos^3 u}$$

18. Solve $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = \log x \sin(\log x)$.

19. If $f(z)$ is an analytic function with constant modulus. Show that $f(z)$ is constant.

20. Expand $\sin z$ in a Taylor's series about $z = 0$ and determine the region of convergence.

21. Test for convergence of the series:

(i) $\sum_{n=1}^{\infty} \frac{n!}{(n^n)^2}$

(ii) $\frac{1}{\log 2} - \frac{1}{\log 3} + \frac{1}{\log 4} - \frac{1}{\log 5} + \dots$

22. By changing the order of integration of $\int_0^{\infty} \int_0^{\infty} e^{-xy} \sin px \, dx \, dy$, show that $\int_0^{\infty} \frac{\sin px}{x} \, dx = \frac{\pi}{2}$.