INDIAN MARITIME UNIVERSITY
(A Central University, Government of India)

May/June End Semester Examinations
B.Sc. (Nautical Science) Second Semester
(AY 2009 – 2013 batches only)

Applied Mathematics -III(UG21T1202)

Date : 07.06.2017 — Maximum Marks: 100
Time: 3 Hrs — Pass Marks : 40

Note: Attempt any five questions. All questions carry equal marks.
Use of Scientific Calculator is allowed.

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1. a. Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$

   b. Prove $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$

   c. Find the value of $J_{1/2}(x)$

(7 + 7 + 6 marks)

2. a. Prove Rodrigue's formula that

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$$

   b. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials.

   c. Show that

$$\int_{-1}^{1} x^2 P_{n-1} P_{n+1} dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$$

(7 + 7 + 6 marks)

3. a. A string is stretched and fastened to two points l apart Motion is started by displacing the string in the form $y = a\sin\left(\frac{\pi x}{l}\right)$ from which it is released at time $= 0$. Show that the displacement of any point at a distance x from one end at time t is given by

$$y(x,t) = a \sin\left(\frac{\pi x}{l}\right) \cos\left(\frac{\pi ct}{l}\right)$$

b. A tightly stretched string of length *l* with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $V_0 \sin^3 \frac{\pi x}{l}$. Find the displacement of $y(x,t)$

(10 + 10 marks)

4. a. Find the solution of the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.

b. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(x,0) = 3 \sin n\pi x, u(0,t) = 0$ and $u(1,t) = 0$ where $0 < x < 1, t > 0$

(10 + 10 marks)

5. Find the Laplace transform for the following functions.

a. $L\ (\sin 2t\ cost\ 3t)$

b. $L\ (e^{-3t} \sin 5t \sin 3t)$

c. $L\left\{t \int_0^t \frac{e^{-t} \sin t}{t}\ dt\right\}$

d. $L\ \left\{\frac{\cos 2t - cost\ 3t}{t}\right\}$

(3.5 + 3.5 + 3.5 + 3.5 marks)

6. Find the inverse Laplace transforms for the following functions.

a. $L^{-1}\left(\frac{S+2}{S^2 - 4s + 13}\right)$

b. $L^{-1}\left[\left(\frac{2S-5}{4s^2 + 25}\right) + \frac{4S-18}{9-S^2}\right]$

c. $L\left(\tan^{-1}\left(\frac{2}{S^2}\right)\right)$

d. $L^{-1}\left(\log\left(\frac{S+1}{S-1}\right)\right)$

(3.5 + 3.5 + 3.5 + 3.5 marks)

7. a. If $w = \phi + i\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2+y^2}$ determine the function ϕ.

b. Evaluate $\int_c \frac{e^{2z}}{(z+1)^4}\,dz$, where c is the circle $|z| = 2$, using Cauchy's integral formula.

(7 + 7 marks)

8. a. Evaluate

$$\int_c \frac{z^3}{(z-1)^4(z-2)(z-3)}\,dz$$

where c is the circle $|z| = 2.5$ using Cauchy's Residue theorem.

b. Evaluate

$$\int_c \frac{(2z-1)dz}{z(z+2)(2z+1)}$$

where c is $|z| = 1$ using Cauchy's Residue theorem.

(7 + 7 marks)

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