

Indian Maritime University
(A Central University, Govt of India)
End Semester Examinations – June 2024
Programme Name: B Tech (ME)
Semester: Second
Subject Code: UG11T4201
Subject Name: Mathematics II

Date: 27.05.2024

Max Marks: 70

Duration: 03 Hrs

Pass Marks: 35

General Instructions

- (i) All Sections (A, B & C) are to be attempted.
- (ii) Options, if any, are specified in respective section.

Section A

Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable.

1. The curve passing through (0,1) and satisfying $\sin\left(\frac{dy}{dx}\right) = c'$ is

- a) $\cos\left\{\frac{(y-1)}{x}\right\} = c'$
- b) $\sin\left\{\frac{(y-1)}{x}\right\} = c'$
- c) $\cos\left\{\frac{x}{(y-1)}\right\} = c'$
- d) $\sin\left\{\frac{x}{(y-1)}\right\} = c'$

2. The complementary function of $(D^3 + D^2 - D - 1)y = 0$ is

- a) $c_1e^x - (c_2 + c_3x)e^{-x}$
- b) $c_1e^{-x} + (c_2 + c_3x)e^x$
- c) $c_1e^x + (c_2 + c_3x)e^{-x}$
- d) $c_1e^{-x} + (c_2 + c_3x)e^x$

3. The number of arbitrary constants in the particular solution of a differential equation of third order is:

- a) 3
- b) 2
- c) 1

d) 0

4. The differential equation $(x + x^8 + ay^2)dx + (y^8 - y + bxy)dy$ is exact if

a) $a = b$ b) $a = 2, b = 3$ c) $b = 2a$ d) $b = a/2$

5. The inverse Laplace Transform of $\frac{1}{s^2+9}$ is

a) $\frac{1}{3} \sin 3t$ b) $\frac{1}{3} \cos 3t$ c) $\sin^2 3t$ d) $\sin 3t$

6. If $L\{f(t)\} = \bar{f}(s)$, then $L^{-1}\left\{\frac{1}{(s^2+1)^2}\right\}$ is equal to

a) $\frac{1}{2}(\sin t - t \cos t)$
b) $\frac{1}{2}(\sin t + t \cos t)$
c) $\frac{1}{2}(\sin t - \cos t)$
d) $\frac{1}{2}(\sin t + \cos t)$

7. The partial differential equation of the given function $Z = ax + a^2y^2 + b$ by eliminating the arbitrary constant is

a) $q = 2p^2y$
b) $p = 2q^2y$
c) $z = px + qy + pq$
d) $pq = 4xyz$

8. The value of b_1 of the Fourier series expansion for the function $f(x) = x \sin x, 0 < x < 2\pi$

a) 2π
b) 3π
c) $-\pi$
d) π

9. The value of $\oint_c \frac{z-3}{z^2+2z+5} dz$, where c is the circle $|z| = 1$ is

a) 1
b) 0
c) 3
d) 2

10. For the function $\frac{\sin z}{z^3}$ of a complex variable the point $z=0$ is

a) a pole of order 4

- b) a pole of order 3
- c) a pole of order 2
- d) not a singularity

Section B

Five Questions of 02 Marks each

11. Find a_0 of the Fourier series expansion for the following function

$$f(x) = |x|, -\pi < x < \pi$$

12. Find $L\left\{\frac{\sin at}{t}\right\}$, given that $L\left\{\frac{\sin t}{t}\right\} = \tan^{-1}\left\{\frac{1}{s}\right\}$

13. Find r , if x^r is an integrating factor of the equation

$$(x+y^3)dx + 6xy^2dy = 0$$

14. Solve : $(x+1)\frac{dy}{dx} + 1 = 2e^{-y}$

15. Form the partial differential equation by eliminating the function f from the relation $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$.

Section C

Seven Questions of 10 Marks each of which any 05 questions to be answered.

16 a) Find the Fourier series for

$$f(x) = x \quad \text{for} \quad 0 \leq x \leq 1$$

$$= 1-x \quad \text{for} \quad 1 \leq x \leq 2$$

(5 marks)

16 b) Find the Fourier series of x^2 in $(-\pi, \pi)$. Use Parseval's identity

$$\text{Prove that } \frac{\pi^4}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \dots \dots \dots$$

(5 marks)

17 a) Solve $x^4 \frac{dy}{dx} + x^3y + \operatorname{cosec}(xy) = 0$

(5 marks)

17 b) Solve $(D^4 + 2D^2 + 1)y = x^2 \cos x$, where $D \equiv \frac{d}{dx}$

(5 marks)

18a) Solve by using the method of variation of parameters

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \sin x$$

(5 marks)

18 b) Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos \log(1+x)$

(5 marks)

19a) Find the particular integral of $y = \frac{1}{D^2 - 5D + 16} x^2 e^{3x}$ (4 marks)

19b) Solve by separation of variables $u_{xx} - u_y = 0$ (6 marks)

20a) Find $L^{-1}\left\{\frac{1}{2} \log \frac{s^2 + b^2}{s^2 + a^2}\right\}$ (5 marks)

20b) Use unit step function to evaluate (5 marks)

$$f(t) = \begin{cases} t, & 0 < t < 2 \\ 2, & t > 2 \end{cases}$$

21a) Using convolution theorem to find $L^{-1}\left\{\frac{s}{(s^2 + 4)^2}\right\}$ (5 marks)

21b) Solve the equation using Laplace Transform

$$\frac{d^2x}{dt^2} + 2 \frac{dx}{dt} + 5x = e^{-t} \sin t, \quad x(0) = 0, \quad x'(0) = 1 \quad (5 \text{ marks})$$

22 a) Determine the analytic function whose real part is $(y + e^x \cos y)$ (5 marks)

b) Evaluate : $\oint_C \frac{e^{2z}}{(z+1)^4} dz$, where C is the circle $|z| = 2$ (5 marks)