

**Indian Maritime University**  
**(A Central University, Govt of India)**  
**End Semester Examinations – December 2022**  
**Programme Name: B Sc (NS)**  
**Semester: I**  
**Subject Code: UG21T5102**  
**Subject Name: Mathematics**

Date: 26.12.2022

Max Marks: 70

Duration: 03 Hrs

Pass Marks: 35

General Instructions

- (i) All Sections (A, B & C) are to be attempted.
- (ii) Options, if any, are specified in respective section.
- (iii) Scientific Calculator is permitted.

Section A

Objective type questions/Fill in the blanks(01 Marks Each)

1. Matrix has a value. This statement

- a) is always true    b) depends upon the matrices    c) is false

2. The sum of the squares of the eigen values of  $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$  is .....

- a) 10    b) 100    c) 38

3. If  $u = x^y$  then  $\frac{\partial u}{\partial y}$  is .....

- a) 0    b)  $yx^{y-1}$     c)  $x^y \log x$

4. The sum of the three angles must lie between  $180^\circ$  and  $540^\circ$ . True or False

5. A great circle is .....

6. The first derivative of  $e^{-x}x^3$  is .....

7. If an error of 1% is made in measuring its length and breadth the percentage error in the area of a rectangle is .....

- a) 0.2%    b) 0.02%    c) 2%    d) 1%

8. The value of  $\beta(2,1) + \beta(1,2)$  is .....

- a) 1    b) 2    c) 0



9. To change cartesian coordinates  $(x, y, z)$  to spherical polar coordinate  $(r, \theta, \phi)$ ;  $dx dy dz$  is replaced by .....
10. If the directional derivative of  $f = ax + by + cz$  at  $(1, 1, 1)$  has maximum magnitude 4 in direction parallel to x axis then the values of a, b, c are .....
- a)  $(-2, 2, 2)$     b)  $(2, -2, 2)$     c)  $(2, 2, -2)$

### Section B

#### Short Answer Type Questions(02 Marks Each)

11. If  $u = \sin^{-1}\left(\frac{x+2y+3z}{x^8+y^8+z^8}\right)$  find the value of  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$
12. In spherical triangle PQR,  $p = 62^\circ 10.1'$ ,  $q = 111^\circ 35.2'$ ,  $r = 63^\circ 33'$  Calculate P.
13. If  $u\vec{F} = \nabla v$ , where u, v are scalar fields and  $\vec{F}$  is a vector field, show that  $\vec{F} \cdot \text{curl } \vec{F} = 0$
14. Change the order of integration and hence prove  $\int_0^\infty \int_x^\infty \frac{1}{y} e^{-y} dx dy = 1$
15. Determine whether the set  $\{t^2+2t-3, t^2+5t, 2t^2-4\}$  of vectors is linearly independent.

### Section C

#### Answer five out of seven questions(10 Marks Each)

16. a) Define quadrantal spherical triangles and mention any two properties of the same (5)
- b) In spherical triangle PQR,  $PQ = 52^\circ 11'$ ,  $Q = 69^\circ 47'$  and  $QR = 90^\circ$  Calculate P, R and PR (5)
17. a) Define spherical triangle and its properties (5)
- b) In spherical triangle WXY,  $W = 88^\circ 24.5'$ ,  $x = 98^\circ 10'$ ,  $y = 100^\circ 09'$  find w and X. (5)
18. a) State Leibnitz' theorem (3)
- b) If  $y = (\sin^{-1}x)^2$  show that  $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - x^2y_n = 0$ . hence find  $(y_n)_0$  (7)
19. a) If  $u = \tan^{-1} \frac{x^3+y^3}{x-y}$  prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$  (4)
- b) Find the extreme points of the function  $z = x^3 + y^3 - 3axy$  (6)
20. a) Evaluate in terms of gamma function, the integral  $\int_0^\infty e^{-x^4} dx$  (5)
- b) Find by double integration the areas enclosed by the curve

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$$a^2y^2 = x^3(2a-x)$$

(5)

21.a) Find the directional derivative  $f = (xx^2 + y^2 + z^2)^{-1/2}$  at the point  $P(3,1,2)$  in the direction of the vector  $yx\mathbf{i} + zx\mathbf{j} + xy\mathbf{k}$

(5)

b) If  $\vec{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$  and  $r = |\vec{r}|$  show that  $\text{div}\left(\frac{\vec{r}}{r^3}\right) = 0$

(5)

22. If  $A = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$  find eigen values and eigen vectors of

a)  $A^2 - 2A + I$     b)  $A^2$     c)  $4A^{-1}$

(10)

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22 ✓  
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