Indian Maritime University

(A Central University, Govt of India)

End Semester Examinations – December 2023

Programme Name: B Tech (NAOE)

Semester: I

Subject Code: UG12T2101

Subject Name: Engineering Mathematics-I

Date: 19.12.2023	Max Marks: 70
Duration: 03 Hrs	Pass Marks: 35

General Instructions

(i) All Sections (A, B & C) are to be attempted.

Section A

Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable.

1. For $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 3e^{2x}$, the particular integral is
   (a) $\frac{1}{15} e^{2x}$
   (b) $\frac{1}{5} e^{2x}$
   (c) $3e^{2x}$
   (d) $c_1e^{-x} + c_2e^{-3x}$
2. If $f(x, y, z) = x^3y^2z - 2xz + 3xy^2$ then f_y =........... three
   (a) $\frac{1}{15} e^{2x}$
   (b) $\frac{1}{5} e^{2x}$
   (c) $3e^{2x}$
   (d) $c_1e^{-x} + c_2e^{-3x}$
3. The reciprocal of the curvature of the curve at any point 'P' is called
   (a) centre of curvature
   (b) circle of curvature
   (c) radius of curvature
   (d) chord of curvature
4. For the function $f(x) = x + \frac{1}{x}$, $x \in [1, 3]$, the value of c for mean value theorem is
   (a) 1
   (b) $\sqrt{3}$
   (c) 2
   (d) none of these

5. Infinite series $1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+ \ldots\ldots$ is

(a) divergent
(b) convergent
(c) oscillatory
(d) none of these

6. By changing the order of integration, $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}$ dy dx the limits of integration becomes

(a) x: 0 to a; y: 0 to ∞
(b) x: y to ∞; y: 0 to x
(c) x: 0 to y; y: 0 to ∞
(d) x: 0 to ∞; y: 0 to y

7. The order of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+y^4=e^{-x}$ is

(a) 1
(b) 2
(c) 3
(d) none of the above

8. $\lim_{x\to 1}\frac{(x^2-1)}{(x-1)}$ is equal to

(a) ∞
(b) 0
(c) 2
(d) 1

9. If the principal part contains an infinite number of non-zero terms of (z-a) then z=a is known as

(a) Poles
(b) Isolated Singularity
(c) Essential Singularity
(d) Removable Singularity

10. Which of the following functions would have only odd powers of x in its Taylor series expansion about the point x = 0?

(a) $\sin(x^3)$
(b) $\sin(x^2)$
(c) $\cos(x^3)$
(d) $\cos(x^2)$

Section B

Five Questions of 02 Marks each

11. Evaluate $\int_0^{\pi/2}\int_{a(1-\cos\theta)}^{a} r^2\, dr d\theta$.

12. Determine all the number(s) c which satisfy the conclusion of Rolle's Theorem for $f(x) = x^2 - 2x - 8$ on $[-1, 3]$.

13. Find the particular integral of $(D^2 - 6D+13)y = 8\, e^{3x}\sin(2x)$.

14. Determine the analytic function w=u+iv, if $v = \log(x^2+y^2) + x - 2y$.

15. Test the convergence of the following series: $\frac{3}{1}+\frac{4}{8}+\frac{5}{27}+\frac{6}{64}+\ldots$

<u>Section C</u>

Seven Questions of 10 Marks each of which any 05 questions to be answered.

16. Verify Cauchy's Mean value theorem for the functions f(x) and f′(x) in (1,e) given f(x) = log x.

17. Find the Taylor's and Laurent's series which represents the function f(z) = $\frac{z^2-1}{(z+2)(z+3)}$ in the regions:

(a) |z|<2

(b) 2<|z|<3

18. Evaluate $\int_{1-i}^{2+3i}(z^2+z)\,dz$ along the line joining the points (1,-1) and (2, 3).

19. Change the order of integral in $\int_0^1\int_{x^2}^{2-x} xy\,dxdy$ and hence evaluate the double integral.

20. Apply Lagrange's Mean value theorem to the function f(x) = log x in [a, a+h] and determine θ in terms of a and h. Hence, deduce that $0 < \frac{1}{\log(1+x)} - \frac{1}{x} < 1$

21. Solve the differential equation: $(D^2+3D+2)y = x\,e^x \sin(x)$.

22. If $u = x^3 \sin^{-1}\left(\frac{y}{x}\right) + x^4 \tan^{-1}\left(\frac{y}{x}\right)$, find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ at x=1, y =1.