Indian Maritime University

(A Central University, Govt of India)

Supplementary Examinations – March/April 2025

Programme Name: B Tech (ME)

Semester: First

Subject Code: UG11T4101

Subject Name: Mathematics I

Date: 03.03.2025	Max Marks: 70
Duration: 03 Hrs	Pass Marks: 35

Section A **(10X1=10 Marks)**

Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable.

1. If u=e^{xyz} then $\frac{\delta^3 u}{\delta x \delta y \delta z}$ at (1,1,1) is
   a) 5e
   b) 4e
   c) 2e
   d) 3e

2. The gradient of a scalar function $\phi(x, y, z)$ is
   a) $\frac{\partial^2 \phi}{\partial x^2} i + \frac{\partial^2 \phi}{\partial y^2} j + \frac{\partial^2 \phi}{\partial z^2} k$
   b) $\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} + \frac{\partial \phi}{\partial z}$
   c) $\frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k$
   d) $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2}$

3. If $\vec{F} = f_1(x, y, z)\vec{i} + f_2(x, y, z)\vec{j} + f_3(x, y, z)\vec{k}$ is a vector function, then the divergence of F is

a) $\frac{\partial^2 f_1}{\partial x^2} i + \frac{\partial^2 f_2}{\partial y^2} j + \frac{\partial^2 f_3}{\partial z^2} k$

b) $\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$

c) $\frac{\partial f_1}{\partial x} i + \frac{\partial f_2}{\partial y} j + \frac{\partial f_3}{\partial z} k$

d) $\frac{\partial^2 f_1}{\partial x^2} + \frac{\partial^2 f_2}{\partial y^2} + \frac{\partial^2 f_3}{\partial z^2}$

4. If A and B are symmetric matrices of the same order, then

a) AB is a symmetric matrix
b) A-B is skew symmetric matrix
c) AB-BA is a skew symmetric matrix.
d) AB+BA is a symmetric matrix

5. The matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a

a) Diagonal matrix
b) Symmetric matrix
c) Skew-symmetric matrix
d) Hermitian matrix

6. If x=uv and y=$\frac{u+v}{u-v}$, then the value of the Jacobian $\frac{\delta(x,y)}{\delta(u,v)}$ is given by

a) $\frac{4uv}{(u-v)^2}$
b) $\frac{4uv}{(u+v)^2}$
c) $\frac{4uv}{(u-v)^3}$
d) $\frac{4uv}{(u+v)^3}$

7. If for non zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b}$ is a unit vector and $|\vec{a}| = |\vec{b}| = \sqrt{2}$,

then angle θ between vectors $\vec{a}$ and $\vec{b}$ is

a) $\frac{\Pi}{2}$

b) $\frac{\Pi}{3}$

c) $\frac{\Pi}{6}$

d) $-\frac{\Pi}{2}$

8. If Rank(A)=2 and Rank(b)=3, then Rank (AB) is
   a) 6
   b) 5
   c) 3
   d) Data inadequate

9. Stationary point is a point where a function f(x,y) have
   a) $\frac{\delta f}{\delta x} = 0$
   b) $\frac{\delta f}{\delta x} = 0$ & $\frac{\delta f}{\delta y} = 0$
   c) $\frac{\delta f}{\delta y} = 0$
   d) $\frac{\delta f}{\delta x} > 0$ & $\frac{\delta f}{\delta y} < 0$

10. For function f(x,y) to have maximum value at (a,b) is
   a) rt-$s^2 > 0, r < 0$
   b) rt-$s^2 > 0, r > 0$
   c) rt-$s^2 < 0, r < 0$
   d) rt-$s^2 < 0, r > 0$

Section B

Five Questions of 02 Marks each (5X2=10 Marks)

11. If $y = \frac{x+3}{(x+1)(x+2)}$, find y_n

12. If u=xy-yz-zx, v=$x^2+y^2+z^2, w = x+y-z$, determine whether they are functionally related or not, if so find the relationship between them.

13. Find the angle between the tangents to the curve $\vec{r} = t^2\vec{i} + 2t\vec{j} + 3\vec{k}$ at the point t=$\pm$1.

14.Evaluate $\lim_{x \to \frac{\pi}{2}} (sinx)^{\tan x}$

15.Expand $e^{\sin x}$ by Maclaurin's series up to the term containing x^4.

Section C

Seven Questions of 10 Marks each of which any 05 questions to be answered.

16 a) If $sin^{-1}y = 2\log(x+1)$, (5)
prove that $(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$

16b) Show that by Lagrange multiplier method if the perimeter of a triangle is constant, the triangle has maximum area when it is equilateral. (5)

17 a) If $u = \tan^{-1}\frac{\sqrt{x^3+y^3}}{\sqrt{x}+\sqrt{y}}$, show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -2\sin^3 u \cos u$$ (5)

17 b) If $z = f(x,y), where\ x = e^u\cos v,\ y = e^u\sin v$,

Show that $\left(\frac{\delta f}{\delta x}\right)^2 + \left(\frac{\delta f}{\delta y}\right)^2 = e^{-2u}\left\{\left(\frac{\delta z}{\delta u}\right)^2 + \left(\frac{\delta z}{\delta v}\right)^2\right\}$ (5)

18a) Prove that $\sqrt{\pi}\Gamma(2m) = 2^{2m-1}\Gamma(m)\Gamma(m+\frac{1}{2})$ (5)

18b) Find by double integration, the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about the y-axis. (5)

19a) Change the order of integration in $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2+y^2}$ and hence evaluate the same. (5)

19b) Evaluate $\iiint_V (x^2+y^2+z^2)\,dx\,dy\,dz$ where

v=(0≤x≤1, 0≤y≤1, 0≤z≤1) (5)

20 a) A vector field is given by $\vec{A}=(x^2+xy^2)\vec{i}+(y^2+x^2y)\vec{j}$. Show that the field is irrotational and find the scalar potential. (5)

20 b) Find the directional derivatives of the function f=$x^2-y^2+2z^2$ at the point P(1,2,3) in the direction of the line PQ where Q is the point (5,0,4).
In what direction it will be maximum? Find also the magnitude of this maximum. (5)

21a) Reduce the following matrix into its normal form and hence find the rank.

$$A=\begin{bmatrix}8 & 1 & 3 & 6\\ 0 & 3 & 2 & 2\\ -8 & -1 & -3 & 4\end{bmatrix}$$ (5)

21 b) Given that matrix

$$A=\begin{bmatrix}0 & 1+2i\\ -1+2i & 1\end{bmatrix},$$

show that $(I-A)(I+A)^{-1}$ is a unitary matrix, where I is a unit matrix (5)

22a) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix}1 & -2\\ -5 & 4\end{bmatrix}$ (5)

22b) Verify Green's theorem for the function

$\int_c [(xy+y^2)dx+x^2dy)]$, where c is bounded by y=x and y=x^2. (5)