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Indian Maritime University
(A Central University, Govt of India)
Sep/Oct'25 SE

Programme Name: B Tech (ME)
Semester: First
Subject Code: UG11T5102

Subject Name: Engineering Mathematics 1

Date: 02.09.2025
Duration: 03 Hrs
Max Marks: 70
Pass Marks: 35

Section A (10X1 = 10 Marks)

Ten MCQs/Fill in the Blanks of 01 Mark each – Choose the correct answer as applicable.

- The nullity of an invertible matrix is
a) 1 b) 0 c) 2 d) 3
- Consider the following matrix A. Which is the correct statement.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- The matrix A is not a diagonal matrix.
 - The eigen values of matrix are 1, 2 and 4
 - The eigen values of matrix are 1 and 2
 - None of the other options
3. The complementary function of $(D^3 + D^2 - D - 1)y = 0$ is

- $c_1 e^x - (c_2 + c_3 x)e^{-x}$
- $c_1 e^{-x} + (c_2 + c_3 x)e^x$
- $c_1 e^x + (c_2 + c_3 x)e^{-x}$
- $c_1 e^{-x} + (c_2 + c_3 x)e^x$

4. If the function $f(x) = x^2 - 4x + 3$ on the interval $[1, 3]$ satisfies the Rolle's theorem

then the value of c is _____

- 1
- 2
- 3
- 2

5. The number of arbitrary constants in the particular solution of a differential equation of third order is:

- 3
- 2
- 1
- 0

6. Expansion of $\cos x$ in ascending powers of x is _____

- $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$
- $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$
- $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$
- $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$

7. $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dx dy =$

- $\frac{3}{35}$
- $\frac{7}{60}$
- $\frac{4}{49}$
- $\frac{2}{15}$

8. The value of the integral $\int_{-1}^1 \int_0^1 \int_0^1 dx dy dz$ is

- 2
- 0
- 1
- 4

9. The value of the line integral $\int_C (2xy^2 dx + 2x^2 y dy + dz)$ along a path joining the origin and the point (1,1,1) is

- 0
- 2
- 4
- 6

10. Find a unit vector normal to the surface $xy^3 z^2 = 4$ at the point (-1,-1,2).

- $\frac{i+3j+k}{\sqrt{11}}$
- $\frac{i-3j-k}{\sqrt{11}}$
- $\frac{i+3j-k}{-\sqrt{11}}$
- $\frac{i+3j-k}{\sqrt{11}}$

Section B

Five Questions of 02 Marks each

(5X2=10 Marks)

11. Verify the rank and nullity theorem for the matrix $A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 9 \\ 1 & 3 & 4 & 1 \end{bmatrix}$

12. Find nth order derivative of $y = e^{3x} \cos^2 x$

13. If $u = xy - yz - zx$, $v = x^2 + y^2 + z^2$, $w = x + y - z$, determine whether they are functionally related or not, if so find the relationship between them.

14. Evaluate $\int_0^a \int_0^b \int_0^c x^2 y^2 z^2 dx dy dz$

15. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ where $\vec{F} = (3x^2 - 3yz) \mathbf{i} + (3y^2 - 3zx) \mathbf{j} + (3z^2 - 3xy) \mathbf{k}$

Section C

Seven Questions of 10 Marks each of which any 05 questions to be answered. (5X10=50 Marks)

16. a) Test for consistency

$$x + y + z = 6; x + 2y + 3z = 14; x + 4y + 7z = 30 \text{ and solve them. (5 marks)}$$

16. b) Find the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and

(i) Verify Cayley-Hamilton theorem for this matrix.

(ii) Find A^{-1}

(3+2=5 marks)

17. a) Find n^{th} derivative of $y = \frac{1}{(x+2)(x+1)(x-1)^2}$.

(5 marks)

17. b) Solve $\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^2}$

(5 marks)

18. a) If $u = \log(\tan x + \tan y + \tan z)$ show that

(5 marks)

$$\sin 2x \cdot u_x + \sin 2y \cdot u_y + \sin 2z \cdot u_z = 2$$

18. b) By Euler's theorem for homogeneous function

(5 marks)

$$u = \frac{(x^2 + y^2)^m}{2m(m-1)} + x^p \left(\frac{y}{x}\right) + y^q \left(\frac{x}{y}\right)$$

$$\text{Prove that } x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (x^2 + y^2)^m$$

19. a) Change the order of integration and hence evaluate the integral

$$\int_0^1 \int_{\frac{x}{2}}^x \frac{x}{y} dx dy$$

(5 marks)

19. b) Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

(5 marks)

20. a) If $Y = (x^2 - 1)^n$, then show that

$$(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$$

(5 marks)

20. b) Using Lagrange's method of undetermined multipliers find the maximum and minimum distances of the point (3,4,12) from the sphere

$$x^2 + y^2 + z^2 = 4.$$

(5 marks)

21. a) The upward speed $v(t)$ of a rocket at time t is approximated by

$v(t) = at^2 + bt + c$, $0 \leq t \leq 100$ where a , b , and c are constants. It has been found that the speed at times $t = 3$, $t = 6$, and $t = 9$ seconds are respectively, 64, 133, and 208 miles per second. Find the speed at time $t = 15$ seconds. Solve the problem using Gaussian Elimination. (6 marks)

21. b) Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$

(4 Marks)

22. a) Prove that $\vec{A} = (6xy + z^3)\vec{i} + (3x^2 - 2z)\vec{j} + (3xz^2 - y)\vec{k}$ is irrotational.

Find the scalar function $f(x,y,z)$ such that $\vec{A} = \nabla f$ (5 marks)

22. b) Examine whether the given equation is exact or not and hence solve it

$$(x^2 + y^2 + x)dx + xydy = 0$$

(5 marks)