Indian Maritime University

(A Central University, Govt of India)

Supplementary Examinations – March/April 2025

Programme Name: B Tech (ME)

Semester: II

Subject Code: UG11T4201

Subject Name: Mathematics II

Date: 13.03.2025	Max Marks: 70
Duration: 03 Hrs	Pass Marks: 35

General Instructions

(i) All Sections (A, B & C) are to be attempted.
(ii) Options, if any, are specified in respective section.
(iii) Scientific calculator is permitted.

Section A

Multiple choice questions /Fill up the blanks [10x1=10 Marks]

1. The Fourier coefficient b_n for the function

$$f(x) = |x|, \ -\pi < x < \pi$$

A. 1 B. π C. 0 D. $\frac{\pi}{2}$

2. Find the Euler's coefficient a_0 when a function f(x) = x, - π ≤ x ≤ π is expressed as Fourier Series.

A. $-\frac{4(-1)^n}{n\pi}$ B. $\frac{4(-1)^n}{n\pi}$

C. $\frac{2(-1)^n}{n\pi}$ D. 0

3. The differential equation $\left(\frac{dx}{dy}\right)^2 + 5y^{\frac{1}{3}} = x$ is

A. linear of degree 3 B. non-linear of order 1 and degree 6
C. non-linear of order 1 and degree 2 D. linear of degree 2

4. The orthogonal trajectory of xy = c is

A. $x^2-y^2= c'$ B. $x^2+y^2= c'$ C. $x^2-y^2= 2x$ D. $x^2+y^2= 2y$

5. The complimentary function of $y''-2y'+y=xe^x \sin x$ is

A. $c_1 e^x + c_2 e^{-x}$ B. $(c_1 x + c_2) ex$ C. $(c_1 x + c_2) e^{-x}$ D. $(c_1 + c_2)e^x$

6. If $L\{f(t)\} = f(s)$, then $L\{e^{-at}f(t)\}$ is.....

A) $f(s-a)$

B) $f(s+a)$

C) $f(s)$

D) None of these

7. $L^{-1}\left[\frac{1}{(s+a)^2}\right]$ = _____

A. e^{at} B. e^{-at} C. $t\,e^{-at}$ D. $t\,e^{at}$

8. L(sin t cos t)= _____ (fill up the blank)

9. The value of Cauchy's Integral formula $\oint \frac{3z^2+7z+1}{(z-1)}dz$,where c is the circle $|z| = 1/2$ is

A. $2\pi i$ B. 0 C. πi D. $\frac{\pi i}{2}$

10 Which one of the following is true?

a. Only the real part of an analytic function satisfies the Laplacian equation
b. Only the imaginary part of an analytic function satisfies the Laplacian equation
c. Neither the real part nor the imaginary part of an analytic function satisfies the Laplacian equation
d. Both the real and imaginary parts of an analytic function satisfy the Laplacian equation.

Section B

Answer the following [5x2 = 10 Marks]

11. Find a_0 , for $f(x) = \begin{cases} 0 & , -\pi \le x \le 0 \\ \sin x & , 0 \le x \le \pi \end{cases}$.

12. Solve :2 cos x$\frac{dy}{dx}$+ 4y sin x = sin 2x

13. Find : $L\,(t^2e^{-3t})$

14. Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

15. Evaluate using Cauchy's Integral formula $\oint \frac{e^{2z}}{(z+1)^4}dz$,where c is the circle $|z| = 1/2$.

Section C

Answer any 5 out of 7 questions.

16. a) Find the Fourier series expansion for the function $f(x) = 2x - x^2$ in (0, 3). (05)

b) Find the Fourier series of $f(x)=x^2$ in the interval $(0, 2\pi)$. (05)

17. a) Solve $(x^3y^2 + x)\,dy + (x^2y^3 - y)dx = 0$ (05)

b) Solve the given linear differential equation

$2\cos x \frac{dy}{dx} + 4y \sin x = \sin 2x$ (05)

18. a) Solve $(D^2+3D+2)\, y = \sin 2x$ (05)

b) Solve by method of variation of parameters $(D^2 - 6D + 9)y = \frac{e^{3X}}{x^2}$ (05)

19. a) Use Laplace Transform method to solve :

$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^{-t}$, $given\ \ y(0) = y'(0) = 1$

(05)

b) Evaluate $\int_0^\infty e^{-2t}\left(\frac{e^{at} - \cos bt}{t}\right)$ (05)

20. a) Find the Inverse Laplace transforms of $\frac{1}{s^2(s^2+a^2)}$ (05)

b) Using Convolution theorem evaluate $L^{-1}\left[\frac{1}{(s+2)(s+3)}\right]$ (05)

21. a) Evaluate $\oint \frac{(z+3)}{(z+1)(z-2)}\, dz$ where c is the circle $|z| = 3$, using Cauchy Residue Theorem. (05)

b) Find the analytic function whose imaginary part is $e^x \sin y$. (05)

22. a) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region

(i) $|z| < 1$ (ii) $1 < |z| < 2$ using Laurent's series (7)

b) Find the orthogonal trajectory of family of straight lines $y = mx$ passing through the origin. (3)