

Indian Maritime University
(A Central University, Govt of India)
End Semester Examinations – December 2023
Programme Name: B Tech (ME)
Semester: II
Subject Code: UG11T4201
Subject Name: Mathematics II

Date: 13.11.2023

Max Marks: 70

Duration: 03 Hrs

Pass Marks: 35

General Instructions

- (i) All Sections (A, B & C) are to be attempted.
- (ii) Options, if any, are specified in respective section.
- (iii) Use of approved type Scientific Calculator is permitted.

Section A

Fill in the Blanks OR Choose the correct answer as applicable. [10X1=10 Marks]

1. If $f(x)$ is an even function in $(-l, l)$ then the value of $b_n =$ _____

- A. 1
- B. π
- C. 0
- D. $\frac{\pi}{2}$

2. Half range Cosine series of $f(x)$ in $(0, c)$ is

- A. $f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{c}$
- B. $f(x) = \frac{1}{2} a_0$
- C. $f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c}$
- D. None of above

3. $L(\cos at) =$ _____

- A. $\frac{s}{s^2 + a^2}, (s > 0)$
- B. $\frac{1}{s^2 + a^2}, (s > 0)$
- C. $\frac{s}{s^2 - a^2}, (s > 0)$
- D. $\frac{s}{s^2 - a^2}, (s > 0)$

4. The partial differential equation

$$\frac{\partial^2 u}{\partial t^2} - c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = 0 ; \text{ where } c \neq 0 \text{ is known as}$$

- a) Wave equation
- b) 1-dimensional heat equation
- c) 2-dimensional heat equation
- d) Laplace equation

5. The order and degree of given differential equation $\frac{d^2 y}{dx^2} + y \left(\frac{dy}{dx} \right)^3 + e^{3x} = 0$ is _____

- A. order 1 degree 3.
- B. order 2 degree 3
- C. order 2 degree 1
- D. order 1 degree 2

6. What is the general solution of the given differential equation?

$$\frac{d^2 x}{dt^2} + 6 \frac{dx}{dt} + 9x = 0$$

- A. $x = (C_1 + C_2 t)e^{-3t}$
- B. $x = (C_1 + C_2)e^{-3t}$
- C. $x = C_1 + C_2 t e^{-3t}$
- D. $x = C_1 + C_2 e^{-3t}$

7. The inverse Laplace transform of $1/s$ is

- a) t
- b) t^2
- c) e^{-t}
- d) 1

8. The singular point of the function $\frac{1}{4z-z^2}$ are

- A. $z = 0$ and $z = -4$
- B. $z = 0$ and $z = 4$
- C. $z = 4$ and $z = -4$
- D. $z = 0$ and $z = 2$

9. Residue of $\frac{\cos z}{z}$ at $z = 0$ is

- A. 1
- B. -1
- C. 2
- D. 0

10. The integrating factor of the given differential equation is

$$\frac{dx}{dy} + \frac{3x}{y} = \frac{1}{y^2}$$

- A. e^{y^3}
- B. y^3
- C. x^3
- D. $-y^3$

Section B

**Answer the following
[5x2 = 10 Marks]**

11. Obtain the coefficient a_0 of Fourier series for $f(x) = e^x$ in the interval $0 < x < 2\pi$.
12. Solve $\frac{dy}{dx} = e^{-2y}(e^{3x} + x^2)$
13. Find the Laplace transforms of $e^{2t} \cos 4t$
14. Solve $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$
15. Evaluate using Cauchy's Integral formula $\oint \frac{3z^2+7z+1}{(z-1)} dz$,
where c is the circle $|z| = 1/2$.

Section C

Answer any 5 out of 7 questions.

16. a) Express $f(x) = x/2$ as a Fourier series in the interval $(-\pi, \pi)$. (4)
- b) Find the Fourier series of $f(x) = x^2$ in the interval $(0, 2\pi)$.
- Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$ (6)
17. a) Solve $(x^4 - 2xy^2 + y^4) dx - (2x^2y - 4xy^3 + \sin y) dy = 0$ (5)
- b) Solve $\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ (5)
18. a) Solve $(D^2 + 3D + 2)y = \sin 2x$ (5)
- b) Solve $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$ (5)

19. a) Evaluate $\int_0^{\infty} e^{-3t} t^5 dt$ (5)

b) Find $L^{-1}\left[\frac{1}{\{(s-1)(s+2)(s+1)\}}\right]$ (5)

20. a) Find Inverse Laplace transforms of $\frac{1}{s^2(s+5)}$ (5)

b) Using Convolution theorem evaluate $L^{-1}\left[\frac{1}{(s+a)(s+b)}\right]$ (5)

21. a) Evaluate $\int_c \frac{(z+3)}{(z+1)(z-2)} dz$ where c is the circle $|z| = 3$. (5)

b) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region $|z| < 1$ using Taylor's series (5)

22. a) Solve the given partial differential equation by direct integration method

$$\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x \quad (5)$$

b) Form the partial differential equation by eliminating the arbitrary constants from $(x-a)^2 + (y-b)^2 + z^2 = c^2$ (5)