

General Instructions

- (i) All Sections (A, B & C) are to be attempted.
- (ii) Options, if any, are specified in respective section.
- (iii) Scientific calculator is permitted.

Section A

Answer All Questions [10x1=10 Marks]

1. The Fourier coefficient  $b_n$  for the function  $f(x) = |x|, -\pi < x < \pi$ 
  - A. 1
  - B.  $\pi$
  - C. 0
  - D.  $\frac{\pi}{2}$
2. Find the Euler's coefficient  $a_0$  when a function  $f(x) = x, -n \leq x \leq n$  is expressed as Fourier Series.

- A.  $\frac{4(-1)^n}{n\pi}$
- B.  $\frac{2(-1)^n}{n\pi}$
- C.  $\frac{4(-1)^n}{n\pi}$
- D. 0

3. The differential equation  $\left(\frac{dy}{dx}\right)^2 + 5y^2 = x$  is
  - A. linear of degree 3
  - B. non-linear of order 1 and degree 6
  - C. non-linear of order 1 and degree 2
  - D. linear of degree 2

4. The orthogonal trajectory of  $xy = c$  is
  - A.  $x^2 - y^2 = c'$
  - B.  $x^2 + y^2 = c'$
  - C.  $x^2 - y^2 = 2x$
  - D.  $x^2 + y^2 = 2y$

5. The complementary function of  $y'' - 2y' + y = xe^x \sin x$  is
  - A.  $c_1 e^x + c_2 e^{-x}$
  - B.  $(c_1 x + c_2) e^x$
  - C.  $(c_1 x + c_2) e^{-x}$
  - D.  $(c_1 + c_2) e^x$

6. The inverse Laplace Transform of  $\frac{1}{s^2-9}$  is

- a)  $\frac{1}{3} \sinh 3t$
- b)  $\frac{1}{3} \cosh 3t$
- c)  $\frac{1}{3} \sin 3t$
- d)  $\frac{1}{3} \cos 3t$

7.  $L^{-1} \left[ \frac{1}{(s+0)^2} \right] =$

- A.  $e^{at}$
- B.  $e^{-at}$
- C.  $t e^{-at}$
- D.  $t e^{at}$

8.  $L(\sin t \cos t) =$

9. The value of Cauchy's Integral formula  $\oint_C \frac{z^{3z+7z+1}}{(z-1)^4} dz$ , where  $C$  is the circle  $|z| = 1/2$  is

- A.  $2\pi i$
- B. 0
- C.  $\pi i$
- D.  $\frac{\pi}{2}$

10. For the function  $\frac{\sinh z}{z^3}$  of a complex variable the point  $z=0$  is

- a) a pole of order 4
- b) a pole of order 3
- c) a pole of order 2
- d) not a singularity

Section B

Answer the following: [5x2 = 10 Marks]

11. Find  $a_0$  of the Fourier series expansion for the following function  $f(x) = |x|, -\pi < x < \pi$
12. Form the partial differential equation by eliminating the function  $f$  from the relation  $z = (x+y)f(x^2 - y^2)$ .
13. Find Laplace Transforms of  $(\sin t - \cos t)^2$

14. Solve  $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$

15. Evaluate using Cauchy's Integral formula  $\oint_C \frac{e^{2z}}{(z+1)^4} dz$ , where  $C$  is the circle  $|z| = 1/2$ .

Section C

[5x10 = 50 Marks]

Answer any 5 out of 7 questions.

16. a) Find the Fourier series for  $f(x) = x$  for  $0 \leq x \leq 1$   
 =  $1-x$  for  $1 \leq x \leq 2$  (05)

- b) Find the Fourier series of  $f(x) = x^2$  in the interval  $(0, 2\pi)$ . (05)

17. a) Solve  $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos \log(1+x)$  (05)
- b) Solve the given linear differential equation  
 $2 \cos x \frac{dy}{dx} + 4y \sin x = \sin 2x$  (05)
18. a) Solve  $(D^2+3D+2)y = \sin 2x$  (05)
- b) Solve by method of variation of parameters  $(D^2-6D+9)y = \frac{e^{3x}}{x^2}$  (05)
19. a) Find the Laplace transform of  $L\left\{\int_0^t e^x \cdot x^2 dx\right\}$  (05)
- b) Evaluate  $\int_0^\infty e^{-2t} \left(\frac{e^{it}-\cos bt}{t}\right)$  (05)
20. a) Solve the equation using Laplace Transform  
 $\frac{d^2x}{dt^2} + 2 \frac{dx}{dt} + 5x = e^{-t} \sin t, \quad x(0) = 0, \quad x'(0) = 1$  (5 marks)
- (05)
- b) Using Convolution theorem evaluate  $L^{-1}\left[\frac{1}{(s+2)(s+3)}\right]$  (05)
21. a) Evaluate  $\oint \frac{(z+3)}{(z+1)(z-2)} dz$  where  $c$  is the circle  $|z| = 3$ . (05)
- b) Use Cauchy Riemann equation to show that the function  $e^x (\cos y + i \sin y)$  is analytic. Find its derivative. (05)
22. Expand  $f(z) = \frac{1}{(z-1)(z-2)}$  in the region  
 (a)  $|z| < 1$  using Taylor's series  
 (b)  $1 < |z| < 2$  using Laurent's series (5+5)